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ART & MATHEMATICS IN ANTONI GAUDÍ'S ARCHITECTURE: "LA SAGRADA FAMÍLIA"

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Abstract. We shortly discuss the work of the great Catalan Architect Antoni Gaudí – and especially his masterpiece "La Sagrada Família" – in relation with the Geometry used by him therein to create his revolutionary projects, as well as the relations that existed at the turn of XX Century between the development of new ideas in Mathematics and Physics (Riemannian Geometry, Relativity, Fractals, Topology) and the development of new forms of Art (Cubism, Art Nouveau, Modernism). We also discuss the "pseudo-magic square" that decorates the façade of "La Sagrada Família".

Key words. Geometry of Curved Surfaces, Magic Squares

Mathematics Subject Classification: Primary 01A16, 51-03

"L'Architettura, sebbene dipenda dalla Matematica, nulla meno ella è un'arte adulatrice, che non vuole punto per la ragione disgustare il senso: onde sebbene molte regole sue seguano i suoi dettami, quando però si tratta che le sue dimostrazioni osservate siano per offendere la vista, le cangia, le lascia, ed infine contraddice alle medesime; onde non sarà infruttuoso, per sapere quello che debba osservare l'architetto, vedere il fine dell'Architettura, ed il suo modo di procedere".

"Architecture, although it depends on Mathematics, nevertheless it is a flattering Art, that by no means wants to be distasteful: even if many of its rules follow the dictates [of Mathematics], when its observed demonstrations result to be offensive for the sight, it changes them, abandons them and eventually contradicts the same; so that, to know what an Architect should observe, it will not be unfruitful to see the real scope of Architecture, as well as its way of proceeding".

Guarino Guarini, Architettura Civile, Torino 1737, Trattato I, Capitolo I

1 Introduction

1.1 Historical Remarks

Antoni Gaudí (in fact, *Antoni Plàcid Guillem Gaudí i Cornet*) was born in Reus (Cataluña) - or maybe in Riudoms - the 25th of June 1852. He died at the age of 73 years in Barcelona, in 1926, leaving to the posterity his most famous and incomplete work of Architecture: the Basilica Church of ***"La Sagrada Família"*** (*The Sacred Family*) – a fantastic monument that is still under construction (Fig. 1), as a symbol of the heritage that Gaudì left to the town of Barcelona, as well as a monument to the revolution in Architecture that Gaudì left to mankind. In this work, as well as in many of his earlier buildings, Gaudì left an impressive mark that reveals his continuous interest in the role that Mathematics (and more generally the observation of Nature) plays in Art in general and in Architecture in particular; see [1].

Following [2] we might recall that Gaudì was somehow influenced by ***Francesc Llorens i Barba***, who professed a spiritualist doctrine according to which Philosophy was *"complete knowledge"*; something that led Gaudì to follow also the ideas of ***Ernst Haeckel***, who put forward a *"biological concept"* of Universe according to which matter was integrated in Thought rather than in Life. The *"General Morphology of Organisms"* of Haeckel – translated in Spanish and published in Barcelona in 1887 – were certainly a source of inspiration for the fantastic geometric and natural shapes that Gaudì used in his work. As we shall also recall below, Gaudì was moreover a fervent Catholic: he saw the presence of God in all manifestation of Nature (see [2], page 19), to the point that his *"scientific principle"* becomes in fact the counterpart of a *"supranatural revelation"*. According to [2] this is why Gaudì was searching the presence of "beauty" in force lines and funicular polygonals, with the consequence of preferring helicoids and parabolas with respect to circles and squares. A way of thinking peculiar of the difference between *"Classicism"* and the pathos of new *"Modernism"* (see [2], p. 19) about which we shall come later.

Fig. 1 "La Sagrada Familía" by Antoni Gaudí, still under Construction, photo by Marcella Giulia Lorenzi

All his career as an Architect, that made him one of the most renowned Architects of the XIX and XX Century and reserved him an eternal place also in the Ghota of Art & Science, was pervaded in fact by a scrupulous attention to the structure that curvilinear Geometry can offer to Architecture – so anticipating and paving the way to the achievements of other great Architects such as ***Le Corbusier*** and ***Calatrava***, and partly also of ***Lloyd Wright***. The new style that in the twenties eventually assumed the name of *"International Style"* originated in fact from the "*Modern Movement"*, that was in turn an evolution of the *"Catalan Modernisme"* (indeed an important part of the *"Art Nouveau"*, also known as *"Liberty"*). All over the World some Architects begun to develop new solutions aimed at integrating solid traditions (such as, e.g., Gothic) with the new technological possibilities (besides Gaudí we can also mention ***Louis Sullivan*** in Chicago, ***Victor Horta*** in Bruxelles, ***Otto Wagner*** in Wien and ***Charles Rennie Mackintosh*** in Glasgow) – the Catalan Architects ***Lluís Domènech i Montaner*** and ***Josep Puig i Cadafalch*** were the most famous followers of Gaudí. Earlier and precursory ideas are due to ***William Morris***, who in the XIX Century dictated the first principles for a renovation in Architecture; these cultural bases saw, across the turn to XX Century, the flourishing of new ideas originated thanks to the great revolution stimulated by new Science and new Technology. All "personal experiments" of Antoni Gaudí are indeed a genuine expression of the roots of this change (Fig. 2). The "International Style" was aimed at constructing a *"Universal Architecture"*, valid globally and independently of the specific site chosen, while the later *"Organic Architecture"* of Frank Lloyd Wright would instead follow the interpretation of the site, as well as of Space and Time.

Fig. 2 "La Sagrada Familía", photo by Marcella Giulia Lorenzi

1.2 Art Nouveau, Cubism and Science at the Turn of XX Century

One of the most important characteristics of the New Style (i.e., Liberty) resides in its continuous reference to Nature, from which the structural elements are derived and given dynamical undulate contours. The clever use of geometrical structures inspired by such curves would produce shapes in the form of trees and flowers, a reason for which the Style was also called *"Floral"*. Some of the curved lines used became clichés, to be adopted by artists worldwide.

It is not casual that all these changes in perspective do in fact temporally coincide with the great revolutions that Science and Technology had across the Century. As we said in another contribution to these Proceedings [3] in the second half of XIX Century and at the beginning of the XX Century a number of new scientific and technological discoveries changed in fact Mathematics as well as our perception of the World; in particular, we mention in Mathematics the change of vision from flat to curved, from single absolute entities to "manifoldness" (i.e., the property of being global by gluing together local pieces), from continuous to discrete, from the geometry of rigid forms (Euclidean Geometry) to Riemannian Geometry (the revenge of curvature against linearity), as well as Topology as the Geometry of "plastic forms". While in Physics the *Theory of Special Relativity* (1905) and the *Theory of General Relativity* (1915-1916), both formulated by ***Albert Einstein*** in his strenuous efforts to describe on a single base both Electromagnetism and Gravity (see [4],[5]), led to a new conception of simultaneity, to a different awareness of the role that Time plays in our understanding of the World, as well as to the mathematical structure of a new Geometry, in which a continuum and curved *SpaceTime* is at the same time the arena of light propagation and of gravitational forces. SpaceTime did acquire a dynamics of its own and Euclidean Geometry – that dominated until the XIX Century as the unique way of measuring "physical reality" – had to be definitely replaced by non-Euclidean Geometry. Einstein's revolution was in a sense a revenge of Time and Dynamism against Space and Staticity (see [3]). On the artistic side the rigidity of an absolute Space with a fixed Euclidean Geometry had already been destroyed by Impressionism, in the search of more emotional paintings. With Cubism, ***Picasso*** and ***Braque*** introduced the idea that a painting might reflect more specific and single viewpoints on a subject all collected together in a single canvas ([6],[7]), de-facto destroying central perspective and replacing it with a "multicentral perspective" (the artwork becomes a manifold, in which separated spatial perceptions are represented altogether and accurately glued). In a sense, Painting acquires some methodology from Science, becoming a cognitive instrument and addressing directly to intellect without passing through physical impressions; see also [8]).

New conceptions of Space and Time had also begun to pervade Art through *Photography* (even if it had to wait many years to be considered as a form of Art) and also *Cinema* was taking its first steps. Linearity and Staticity were eventually dethroned in favor of Curvature and Velocity. Future begun to be perceived as something less remote and, in 1909, the ***"Futurists"*** had a new artistic vision (see [3],[9],[10]) that aimed at creating a new form of imagery expressing movement (*"All things move, all things run, all things are rapidly changing. A profile is never motionless before our eyes, but it constantly appears and disappears"* – quoted from the *"Futurist Manifesto"*; [9]). ***Guillame Apollinaire***, theorizing Cubism, declared in 1913: *"Today scientists no longer limit themselves to the three dimensions of Euclid. The painters have been led quite naturally, one might say by intuition, to preoccupy themselves with the new possibilities of spatial measurement which, in the language of modern studios, are designated by the term:* ***the fourth dimension****"* (see also [9]).

Worthwhile to mention is also the birth, across the same period, of the further revolutionary idea that continuity and dimension may be different from the notions that resisted since the time of Greeks, through the advent of *"fractals"* (self-reproducing objects at smaller and smaller scales that at the same time show an ordered and a chaotic behaviour; see also the Lecture of D. Velichova in these Proceedings [11]). On purpose or not, it is remarkable that Gaudí used a lot of fractal ideas in his Architecture (see later).

1.3 Gaudí and "La Sagrada Família"

These are precisely the years and the cultural substrate in which Gaudí operated in between the XIX Century and 1926, dead in poverty but destined to become famous for his unique and highly individualistic designs. The years in which Gaudí was – as we said – rather attentive to the new aesthetic developments of Geometry but also to the mysteries of antique Symmetries and of Arithmetic. He lived part of his youth in isolation, and this is considered one of the reasons why he tended to include natural shapes and themes into his later work. He was awarded the title of Architect in Barcelona in 1878 and at that time ***Elies Rogent*** declared in Catalan: *"Qui sap si hem donat el diploma a un boig o a un geni: el temps ens ho dirà" ("Who knows if we have given this diploma to a nut or to a genius…? Only time will tell us"*). As it often happens, time told that he was in fact a genius…! In the same year Gaudí had an acquaintance with ***Eusebi Güell*** - a rich industrialist who eventually become his only real supporter and for whom he would later make many masterpieces of his new Architecture. From 1878 to 1882 he designed for the *Obrera Mataronense* at Mataró and his architectural plans shows his first use of parabolic arches, later replaced by catenaries. The crypt of the Sagrada Família (that had been started by Francisco del Villar in 1882, who abandoned the project in 1883) was completed by Gaudí in between 1884 and 1891. Gaudí was in fact a rather devout Catholic: in his later years he eventually abandoned secular work and devoted the rest of his life to Sagrada Família. He then planned a fantasmagoric constructive project, in which curved Geometry and Fractals dominated alongwith remnants of antique esoteric knowledge. He designed the Sagrada Família to have 18 towers (12 for the 12 Apostles, 4 for the 4 Evangelists, one for the Virgin Mary and one for Christ). The construction was unfortunately slowed down both by personal problems of Gaudí and by Barcelona hard economic times. Also the construction of *"La Colonia Güell"* for his main patron Eusebi Güell ceased altogether (in 1918 Güell did in fact die). Gaudí became introspective and concentrated on his masterpiece, to the point of spending the last few years of his life living in that crypt. On 7 June 1926 Gaudí was run over by a tramway and he died three days later, on 10 June 1926.

Fig. 3 Pinnacles of "La Sagrada Família", photo by Marcella Giulia Lorenzi

As far as the construction of Sagrada Família was progressing the style adopted by Gaudí became more and more fantastic. The four towers – inherited from the (neo)-Gothic Architecture – reminded termite's houses. These towers are terminated by cusps having a precise geometric form, covered by multicolored ceramic tiles, certainly influenced by Cubism (they were in fact finished around 1920); their pinnacles (Fig. 3) are a composition of various intertwined geometrical elements (among which Platonic Solids abound: triangular pyramids, cubes, octahedral; but also spheres and other figures; Fig. 4.a, 4.b and 4.c – see also [12], page 35).

Fig. 4.a Cubic Elements for the Pinnacles of "La Sagrada Familía", in the Museum of the Church, photo by Marcella Giulia Lorenzi

Fig. 4.b Platonic Solids and other Geometrical Shapes for the Pinnacles of "La Sagrada Familía", in the Museum of the Church, photo by Marcella Giulia Lorenzi

Fig. 4.c Elements for the Pinnacles of "La Sagrada Familía", in the Museum of the Church, photo by Marcella Giulia Lorenzi

Many decorations of the Basilica are directly referable to the Art Nouveau and, as such, to the geometric shapes that characterize this new artistic style of the beginning of the XX Century. It has been argued that Gaudí – conscious of the fact the construction of the Church would much probably extend well beyond his death – preferred not to complete the perimeter of the Church but rather dictate its geometric "vertical" structure by completing some of the geometrized towers. After 1940 the construction was retaken by ***Francesc Quintana***, ***Puig Boada*** and ***Lluis Gari***, with sculptures by ***J. Busquets*** and ***Josep Subirachs*** in the façade. The current work are computer assisted; the computer modelling has revealed itself to be extremely useful to give a perfect and coherent geometric shape to the various elements, such as internal pillars and columns (Fig. 5).

Fig. 5 Columns of "La Sagrada Familía", photo by Marcella Giulia Lorenzi

The central naves are characterized by columns in the form of trees and a ceiling that is formed by a tessellation of shapes resembling sunflowers, with a great rotational symmetry (Fig. 6).

Fig. 6 The Vaults of "La Sagrada Familía", photo by Marcella Giulia Lorenzi

It is remarkable that Gaudì did in fact invent a new geometric structure for his columns, a structure *"able to include motion and change"*, much in the spirit of the new age and of Futurism itself (see [12], page 64). Gaudì ideated a column with a "variable transversal shape", i.e. a polygonal shape (either a regular polygon or a starred one) that gradually changes its degree of rotationally symmetry, i.e. it increases gradually its number of sides, passing slowly to an almost circular section and eventually to a circle (Fig. 7). The result was obtained by the intersection of two helicoidal columns, one rotating clockwise and other counterclockwise. At each intersection the number of edges increases with eight (in fact it doubles), while the vertices reduce their size, so that the section is finally turned into something very near to a circle.

Fig. 7 Columns of "La Sagrada Familía", photo by Marcella Giulia Lorenzi

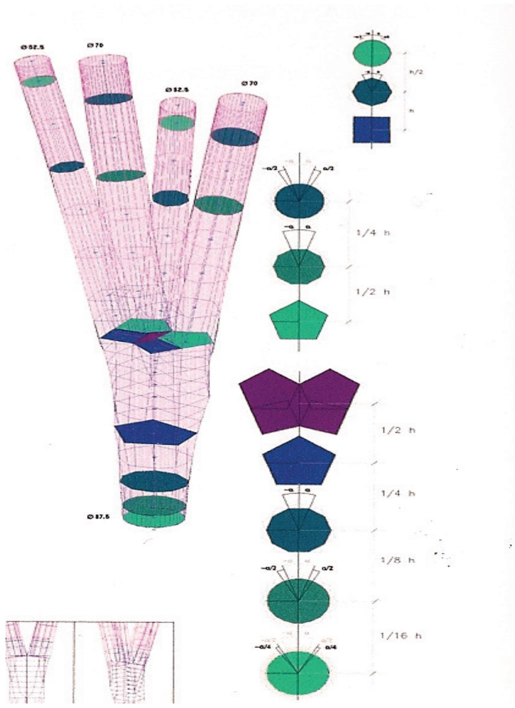

Fig. 8 The Shape of Columns of "La Sagrada Familía", image from [12]

As en example, with such a progression, a square is transformed into an octagon, then in a polygon with 16 sides, and so on as far as the column progresses vertically (Fig. 8). The columns of "La Sagrada Família" present different initial sectional shapes, ranging from 4 to 12 initial sides, that of course increase with height as discussed before. The family of "structural columns" includes also intermediate columns that in turn form the base for smaller columns, in number from 2 to 5, necessary to split the weight of the vault into separate forces. Also these smaller "inverted" columns share the same geometric construction based on the intersection of two helicoids, in which the passage from a polygon to a circle is now obtained from top to bottom.

Fig. 9 Columns resemble Trees, from the Museum of the Church, photo by Marcella Giulia Lorenzi

This clever system not only generates rather interesting geometrical shapes and symmetries, but also allows to mimic an astonishing continuity between lines and surfaces of different columns, exactly as it happens in Nature when passing from the trunk to the stakes of a tree (Fig. 9 - see again [12], page 66). A fully *"natural fractal structure"*, in fact, much in the aforementioned spirit of imitation of Nature. It is impressive how Gaudì was in fact able to obtain such a naturalistic structure by a clever use of Euclidean Geometry, with a fractal process of self-similarity and self-reproduction (Fig. 10). As another example of his creative geometric geniality, we mention that in the "Schools" that stand in the area of the Basilica – designed in 1909 – Gaudí made a clever use of small straight segments to construct curved surfaces. The roof is in fact a conoid with a sinusoidal section (see [12], page 70), again a "natural shape", able to produce a good acoustic and also to be robustly sustained by relatively thin walls (just 9 centimeters for a building of 10 x 20 meters and an eight of 5 meters…!).

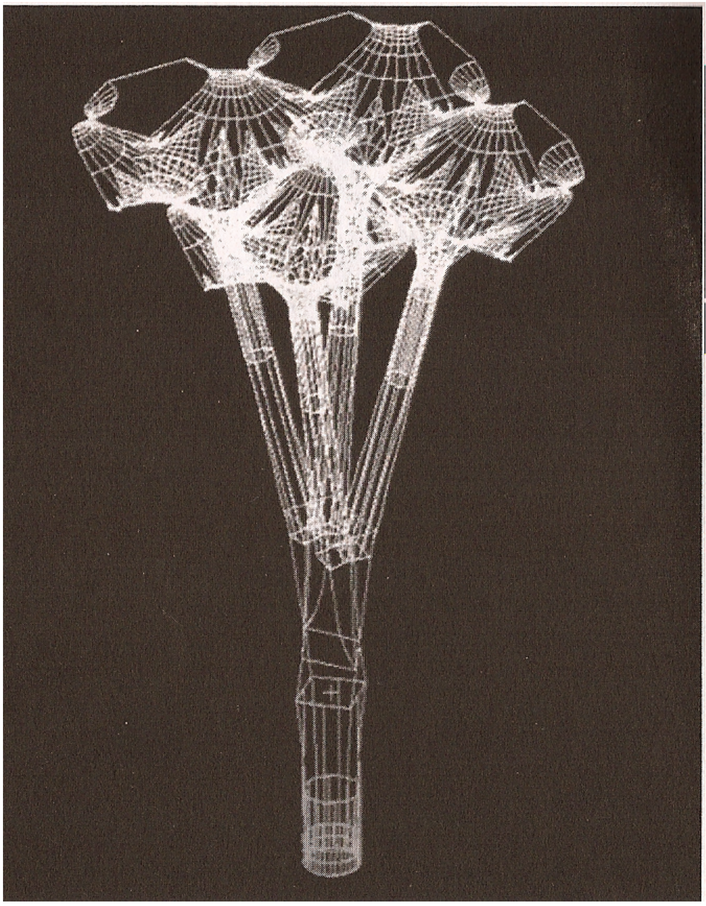

Fig. 10 Computer Design of the Construction of Columns
(a project of Victoria University of Wellington, New Zealand, and Politecnico de Barcelona, Spain)

As we said, "La Sagrada Família" is still under construction (Fig. 11). The central cupola of the Basilica – yet to be terminated – will have a circular shape. It should be remarked that the only existing copy of Gaudí's last recorded blue prints was destroyed in 1938 during the Spanish Civil War and this made it very difficult for his workers to complete the Sagrada Família in the fashion Gaudí most likely would have wished; this masterpiece is now being completed, but differences between the original project of Gaudí and the new additions can be easily recognized.

Fig. 11 "La Sagrada Família" is still under Construction, photo by Marcella Giulia Lorenzi

2 Magic and "Pseudo-Magic Squares"

Worth of mention is the *"pseudo magic square"* of Subirachs that appears in the *"Passion Façade"* (Fig. 12). This numerical table is not a "magic square" in the appropriate sense: a ***"magic square"*** is in fact a (n x n) matrix containing all numbers from 1 to n^2, such that all arrows, all columns and all diagonals give always the same sum, called its *"magic sum"*; this one is in fact a 4 x 4 matrix, in which two numbers are missing (12 and 16) while two other numbers (10 and 14) are repeated twice. Here the sum is always 33, namely the age of Christ. It can be easily shown that this square is not magic, since it is impossible to construct a magic square having 33 as "magic sum": each magic square with 16 entries should in fact have magic sum 34.

Fig. 12 The Pseudo-Magic Square of Subirachis in the Passion Façade, photo by Marcella G. Lorenzi

More generally, in fact, if one has such a square matrix (magic or not) the sum $_n$ of each arrow (and column, and diagonal) should obey the equation

$$n.\sum_n = \frac{1}{2} n^2 \left(n^2 + 1\right)$$

so that

$$\sum_n = \frac{1}{2} n \left(n^2 + 1\right)$$

One has thence $\sum_2 = 5$, $\sum_3 = 15$ and $\sum_4 = 34$. These are, of course, necessary but not sufficient conditions for "magicity".
The "pseudo magic square" of Subirachis can in fact be derived from the following square matrix, that does in fact form a magic square

1	14	15	4
12	7	6	9
8	11	10	5
13	2	3	16

Fig. 13 The Magic Square of Albrecht Dürer (Melancholia)

that appears (modulo a rotation of) in the famous *"Melancholia"* of ***Albrecht Dürer*** (Fig. 13). It is enough to subtract in it 1 from its four entries 15, 12, 11 and 16 (in four independent lines and arrows – Fig. 14) to reduce the sum 34 to 33 and to generate the Subirachis square:

1	14	14	4
11	7	6	9
8	10	10	5
13	2	3	15

Fig. 14 The Pseudo-Magic Square of Subirachis

It has been argued that Subirachis choose this particular square (there are in fact other possibilities) since the sum of the two repeated numbers is 48 (namely, 10+10+14+14), that in the Latin alphabet is also the sum of the places of the four letters I.N.R.I. (namely, 48 = 9+13+17+9). The same sum 33 can be obtained in several other ways (See Fig. 15).

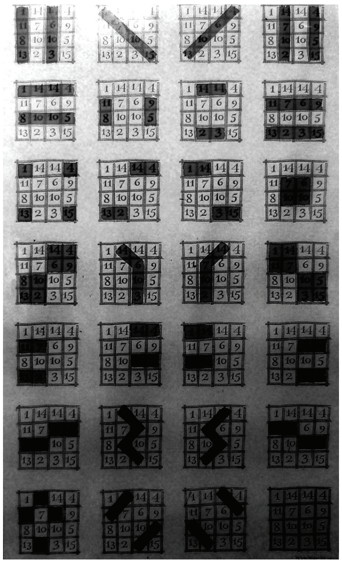

Fig. 15 Summing up to 33 in the Pseudo-Magic Square of Subirachis, from the Museum of the Church, photo by Marcella G. Lorenzi

3 Gaudí and Catenaries

Gaudí begun working in the gothic architectural style (also under the influence of ***Eugene Viollet-le-Duc***) but he soon developed his own distinct sculptural style (irregular and fantastically intricate). For the design of "La Sagrada Família" Gaudí studied and developed a new method of structural calculation based on models involving ropes and small sacks of lead shot (Figg. 16 and 17). The outline of the church was traced on wood and placed on a ceiling, with ropes that hung from the points where columns had to be placed. The sacks of pellets - weighing just a fraction of the weight the real arches would have to support - were hung from each arch formed by the ropes. These ware in fact catenaric arches, as the *Calculus of Variation* dictates. After taking photographs of the resulting model, shot from various angles, turning then upside-down the lines of tension formed by the ropes and weights would eventually indicate the pressure lines of the structure envisaged. In this way Gaudí obtained many natural forms in his work.

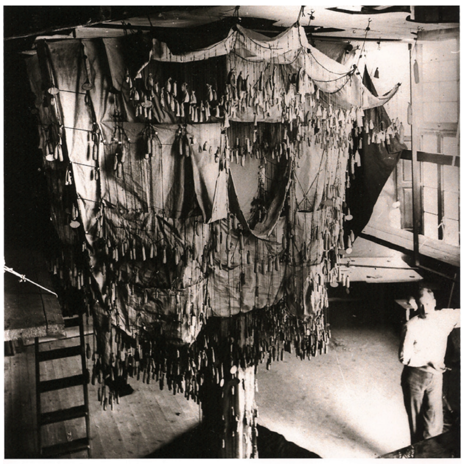

Fig. 16 Archive Photo with the Model chosen for Simulation of Forces

Fig. 17 The Model for the Construction, from the Museum of the Church, photo by Marcella Giulia Lorenzi

In Geometry – as well as in Physics - a “catenary” is a plane curve representing the shape that an idealized hanging chain (or better a rope) assumes when it is supported at its fixed ends and acted upon by gravitational Galilean forces (i.e., weight); see [13]. The name is derived from the Latin word *“catena”*, i.e. "chain". The curve has a U-like shape and is in fact related with the graph of the hyperbolic cosine; it is superficially similar to a parabola, especially in small portions. Its surface of revolution, called “catenoid”, is a minimal surface and is therefore the shape assumed by a soap film bounded by two parallel circles (as it was first proved by ***Euler*** in 1744). ***Galileo*** was the first to define it [14] but he erroneously believed it was just a parabola; this was disproved by ***Joachim Jungius*** (1587–1657) and published posthumously in 1669. The correct equation was in fact derived in 1691 by ***Leibniz***, ***Christiaan Huygens*** and ***Johann Bernoulli***. Huygens first used the term “catenaria” in a letter to Leibniz in 1690. The catenary begun early to be used in the construction of arches (already at the time of pre-Greek and pre-Roman Architecture). In antiquity the curvature of the inverted catenary was in fact intuitively discovered and understood to be useful in the construction of stable arches and vaults; see [15]. Examples may be found in *Taq-i Kisra* in Ctesiphon (Mesopotamia), while Greek and Roman cultures reverted to the (much less efficient curvature of the circle) in circular arches and semi-spherical vaults. It somehow remained in Arab Architecture but in Europe it remained therefore forgotten for long time. It is supposed that its “modern rediscovery” was due to ***Robert Hooke*** - famous for his studies on Elasticity - who discovered it in the context of the rebuilding of St Paul's Cathedral. In 1671 Hooke did in fact announce to the Royal Society that he had solved the problem of the optimal shape of an arch, and in 1675 published an encrypted solution as a Latin anagram in an appendix to his Description of Helioscopes, where he wrote that he had found *"a true mathematical and mechanical form of all manner of Arches for Building"* (see [13]).

The catenary is very important in modern Architecture; it is in fact the ideal curve for an arch that supports only its own weight. When the centerline of an arch follows the curve of an inverted catenary, then the arch is known to endure only pure compression (at a good first approximation), so that no significant torsional moments occur inside the material. When individual pieces form the arch and their contacting surfaces are perpendicular to the curve of the arch, moreover, it is known that no shear forces are present at the contact (again in a good first approximation). No specific buttress is required, since the forces acting on the arch at the two endpoints are tangent to its centerline.

Antoni Gaudí made extensive use of catenary shapes not only in the Sagrada Família but in most of his architectural work. First in the crypt of the Church of *Colònia Güell*, for which he constructed inverted scale models (a technique that worked well for the vaults with a single-curvature). The same technique could not be however used to project the more complex vaults of the nave of the Sagrada Família, that present in fact a double-curvature.

The Cartesian equation of a catenary can be written under the following form

$$y = a \cosh (x/a) = ½[\exp(x/a) + \exp(-x/a)]$$

where *cosh* is the hyperbolic cosine function and *a* is a real constant, that can be interpreted as the ratio between the horizontal component of the tension on the chain (assumed to be homogeneous, so that the tension is constant) and the weight of the chain per length unit.
It is also possible to derive two equations which together define the shape of the curve and the tension of the chain at each single point. See [13].

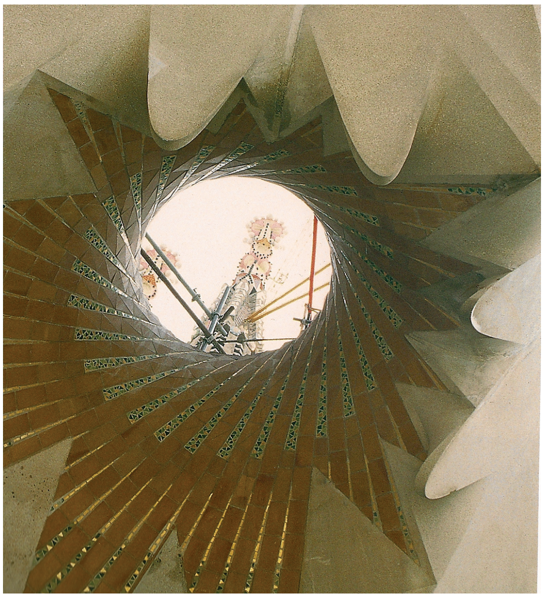

Fig. 18.a Hyperbolic Paraboloids in the Interior of the Vaults of "La Sagrada Família"

4 Final Remarks: Paraboloids, Hyperboloids and Nature in Gaudí

As we already said, throughout his life Gaudí studied natural angles and curves and tried to incorporated them into his sculptured particulars, into mosaics and into the designs of his Architecture. Instead of relying directly on the "simple shapes" of Geometry he rather mimicked Nature by subtly combining them (see our remarks above); in his work rotational surfaces having a peculiar role, such as hyperboloids and paraboloids (Figg. 18.a and 18.b), were in fact borrowed from Nature, so to allow his work to resemble environmental elements. He said once: *"Those who look for the laws of Nature as a support for their new works collaborate with God"*.

For example, an hyperbolic paraboloid (formed by straight lines according to the fact that it is a ruled surface – Fig. 19) was used in the construction of the *"Colonia Güell"* (see [12], page 72); and the use of parabolas can be seen elsewhere (Fig. 20) as well as in various particulars of "La Sagrada Família" (e.g., the columns of the "Passion Façade", the coverings of the naves, the junctions between light openings, the central towers as well as the huge Sacristies). And hyperboloids can be found also in "La Sagrada Família". Gaudí was the first Architect to make a conscious use of such kind of quadrics. Realized again as ruled surfaces, they were used to design and construct some openings between the columns and the vault, with the purpose of giving more light to the interior (see [12], page 74); this was a rather clever choice, able to capture a lot of light and diffuse it in the interior both because of its negative curvature in one direction at the saddle points and because of them being formed by two families of straight lines revolving in two different senses around the circular section of the surface.

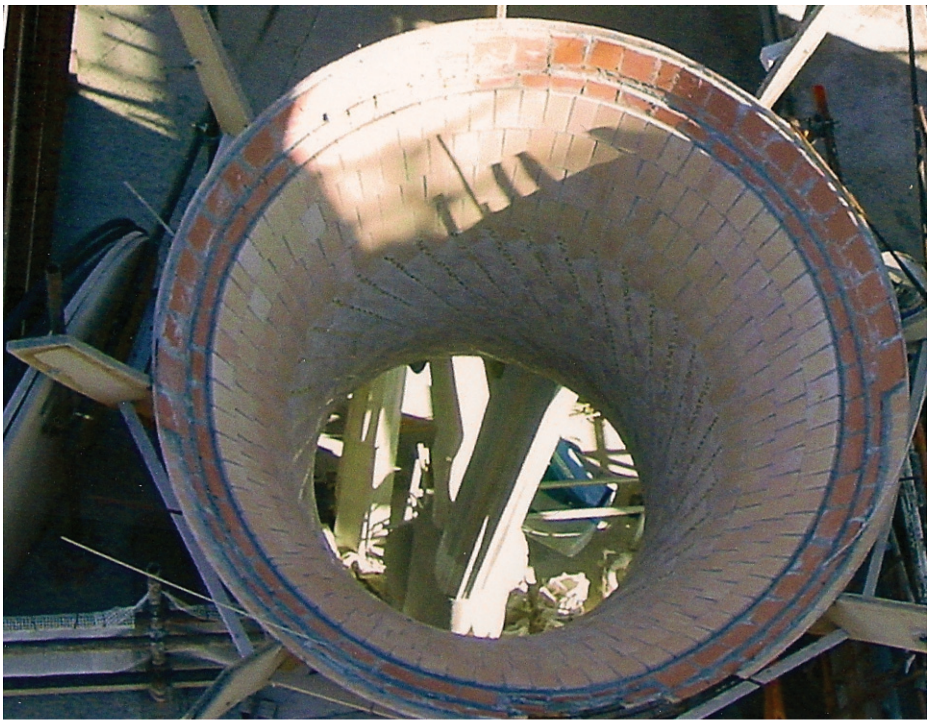

Fig. 18.b Hyperbolic Paraboloids from the Exterior of the Vaults of "La Sagrada Familía"

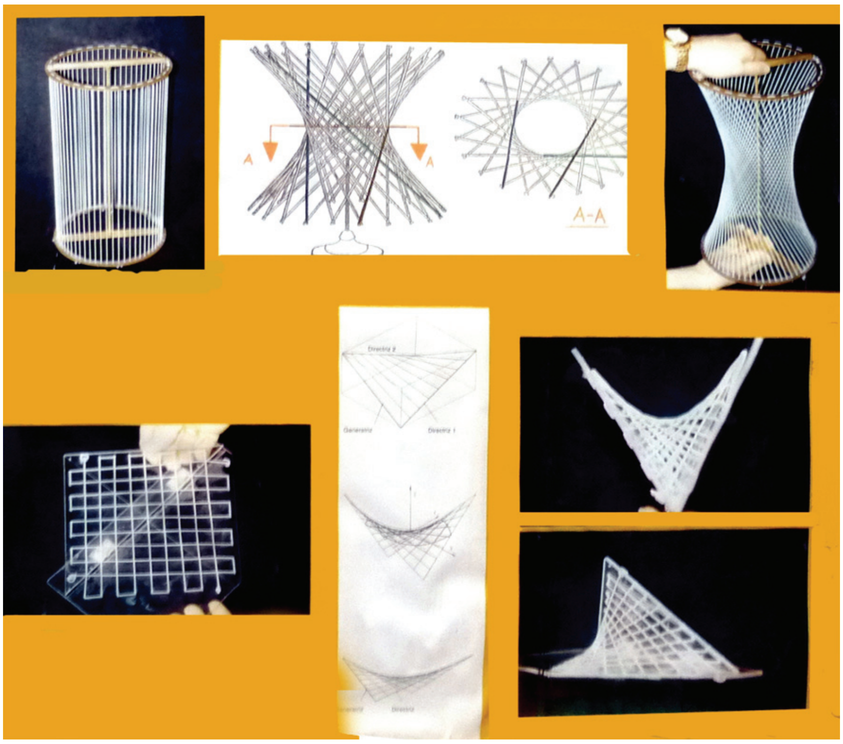

Fig. 19 Designing Hyperboloids and Paraboloids, Museum of the Church, photo by Marcella G. Lorenzi

Fig. 20 Parabolic Vaults in "Casa Batlló"

We like to mention that Gaudí – working in a Liberty style – was in fact implicitly referring also to the structure of Mandala's (see, e.g., our earlier paper [16]). According to [17] a Mandala is *"an organized figure with a centre around which dynamical relations are established with any other* *point of the Mandala, so that an external bound exists that gives perfection to the whole object"*. In [17] a number of Mandalic figures are drawn out of Gaudí's designs; e.g., several ones belonging to *"Park Güell"* and others recurrent in most of Gaudí's decorative work (Fig. 21).

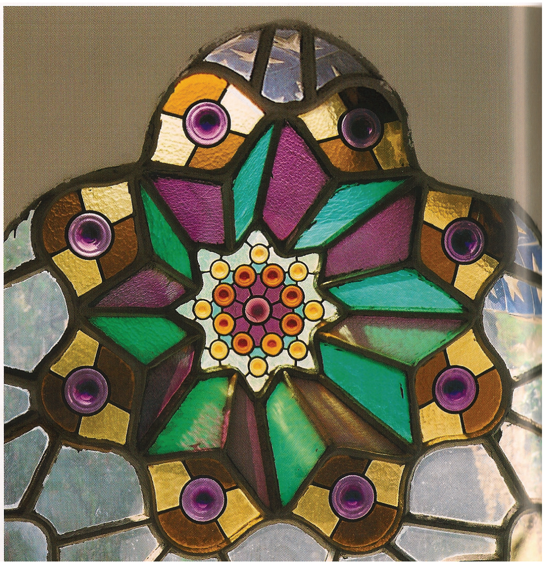

Fig. 21 A Mandala Shaped Vitral in Bellesguard

A further interesting remark concerns the interest towards "geodesics" that Gaudí showed in his celebrated house *"La Pedrera"*, that are particularly evident in the shape of the roof of this work (see [2], page 73) and in its undulate and helicoidal chimneys, but also in its animated façade in stone, with seven curved plies that surround esoteric balconies. The construction lasted from 1906 to 1912; it manifestly shows curved surfaces that gradually pass from plane to ovoidal shapes (so anticipating Arp and Brancusi; see [18]). Not to speak of helicoidal stairs…(Fig. 22).

Fig. 22 Helicoidal Stairs in "La Sagrada Familia", photo by Marcella G. Lorenzi

Just to mention another example, we recall that the central hall of what should have been the house of the family Güell was a fantastic reproduction of Cosmos – a celestial cupola that, according to critics, followed the constructive canons of the cupola of Saint Paul's Cathedral in London (Fig. 23). Gaudí's originality was at first misunderstood and even poked fun at; his work became more famous after his death and he stands now as one of history's most original architects. Gaudí's influence on Culture can be recognized also in the Pokémon film *"The Rise of Darkrai"* [19] as well as in the later work of ***Godey***, designer of the *Space-Time Towers* [20] (which are in fact based on the "Sagrada Família").

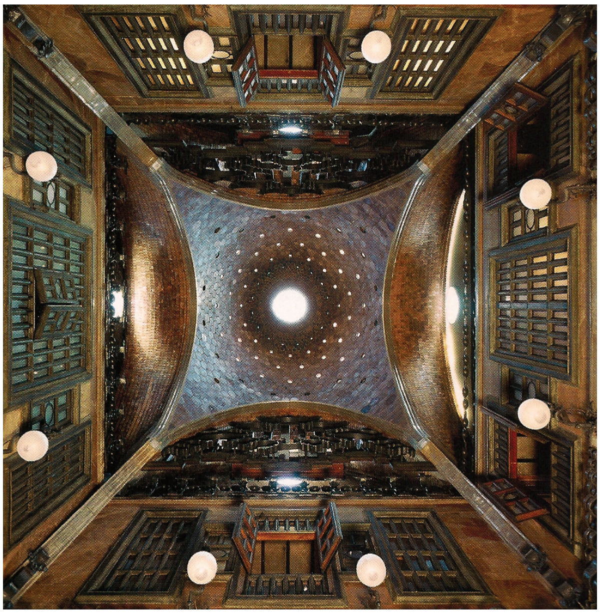

Fig. 23 The "Cosmic" Cupola in "Casa Güell"

This is a list of Gaudí's main Artworks: *Casa Vicens* (1884–1885), *Palau Güell* (1885–1889), the *College of the Teresianas* (1888–1890), the *Crypt of the Church of Colònia Güell* (1898–1916), the *Casa Calvet* (1899–1904), the *Casa Batlló* (1905–1907), *Casa Milà* (better known as *"La Pedrera"*, 1905–1907), *Park Güell* (1900–1914), the *Nativity façade and Crypt of "La Sagrada Família"* (1884-1926, still under completion as of 2010).

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